Circles

Review Exercise Answers

Level-1

Single Choice Correct Only

S1. (C). Since ∠AOC= 2∠ABC and ∠BOD = 2∠BCD, we have:

$$\angle AOC + \angle BOD = 2(\angle ABC + \angle BCD) \quad ...(i)$$

In ΔBEC, using the exterior angle theorem, we have:

$$\angle AEC = \angle ABC + \angle BCD \qquad ...(ii)$$

Using (i) and (ii), we have

$$\angle AOC + \angle BOD = 2\angle AEC$$

Multiple Options may be Correct

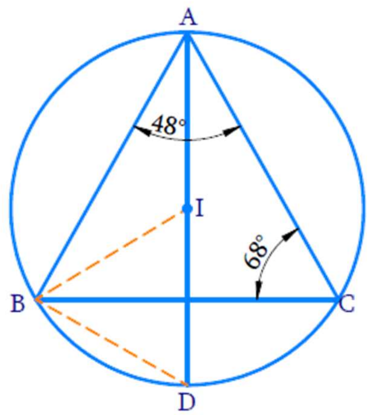
S2. All the four options are correct. We note that

$$\angle OAD = \angle ODA = \frac{1}{2}(180^\circ - \angle a)$$
$$= 90^\circ - \frac{1}{2}\angle a$$

Now,

$$\angle d = \angle CBD = CAD = 90^\circ - \frac{1}{2}\angle a$$
$$\angle c = \angle ACD = \frac{1}{2}\angle AOD = \frac{1}{2}\angle a$$
$$\angle b = \angle BAC = \angle BDC = 90^\circ - \angle c$$
$$= 90^\circ - \frac{1}{2}\angle a$$

S3. (A), (B), (C) and (D). Consider the following figure:



Using the angle sum property in ΔABC, we have:

$$\angle ABC = 180^\circ - (48^\circ + 68^\circ)$$
$$= 64^\circ$$

Now, we make the following observations:

i. $\angle DBC = DAC = \frac{1}{2} \times \angle BAC = 24^\circ$

ii. $\angle IBC = \frac{1}{2} \times \angle ABC = 32^\circ$

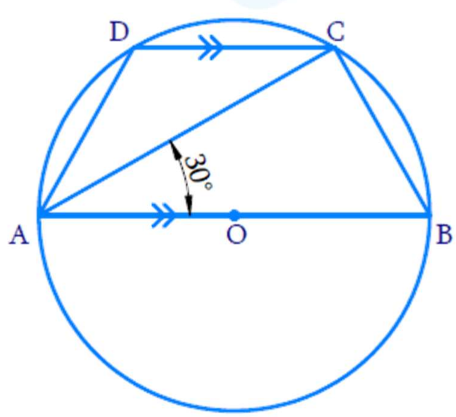
iii. $\angle ADB = \angle ACB = 68^\circ$

iv. $\angle BID = \angle BAI + \angle ABI$

$$= \frac{1}{2} \times \angle BAC + \frac{1}{2} \times \angle ABC$$
$$= \frac{1}{2} \times (48^\circ + 64^\circ)$$
$$= 56^\circ$$

Thus, all the four options are correct.

S4. (B) and (D). Consider the following figure:



We make the following observations:

i. $\angle ACB = 90º$, because the is an angle in a semi-circle

ii. $\angle ABC = 180º – (90º + 30º) = 60º$

iii. Since $\angle ADC$ and $\angle ABC$ are opposite angles of a cyclic quadrilateral, we have:

$$\angle ADC = 180° - \angle ABC$$
$$= 180° - 60° = 120°$$

iv. Since CD || AB, ADC and $\angle DAC$ are supplementary co-interior angles, and so

$$\angle DAB = 180° - \angle ADC$$
$$= 180° - 120° = 60°$$

Thus, $\angle DAC = 60° - 30° = 30°$.

S5. (B), (C) and (D). We make the following observations:

i. $\angle ABE = \frac{1}{2}\angle AOE = 40°$

$\Rightarrow \angle OEB = \angle OBE = 40°$

ii. $\angle BCE = \frac{1}{2}\angle BOE$

$$= \frac{1}{2}(180° - 80°) = 50°$$

iii. $DBC = \angle BCE = 50°$

iv. $\angle DCE + \angle DBE = 180°$

$\Rightarrow \angle DCE = 180° - \angle DBE = 70°$

$\Rightarrow \angle DCB = \angle DCE - \angle BCE$

$=70º – 50º = 20º$

Thus, (A) and (D) are incorrect.

S6. (B), (C) and (D). We make the following observations:

i. $\angle BOC = 2\angle BAC = 40º$

ii. $\angle DCA = \angle CAB = 20º$

iii. $\angle DOA = 2\angle DCA = 40º$

iv. $\angle COD = 180º – (\angle DOA + \angle BOC)$

$= 180º– (40º + 40º)$

$= 100º$

We see that (A) is incorrect.

Integer Answers

S7. We note that $\angle ABC$ is right (angle in a semi-circle). Thus,

$$\angle ACB = 180° - (70° + 90°)$$
$$= 20°$$

Now, in $\angle BCD$, we have (using the angle sum property):

$$x° = 180° - (90° + 20°)$$
$$= 70°$$

S8. We have:

$$x^{o} = \angle BDC = \frac{1}{2}\angle BOC$$
$$= \frac{1}{2}(180^{o} - 150^{o}) = \frac{1}{2}\times 30^{o}$$
$$= 15^{o}$$

S9. Since ∠OBA = ∠OAB = 25º as well, we have:

$$x^{o} = 180^{o} - (25^{o} + 25^{o})$$
$$= 130^{o}$$

S10. Since OB = OC (radii), we have ∠OCB = ∠OBC = 50º. Thus, ∠BDA = ∠BCA = 50º.

S11. We note that ∠CBD = ∠CAD = 40º. Using the angle sum property in ΔBCD, we have:

$$x^{o} = 180^{o} - (40^{o} + 60^{o})$$
$$= 80^{o}$$

S12. We note that ∠BDC = ∠BAC = xº, Now, we use the exterior angle theorem in ΔCDE:

$$130^{o} = 30^{o} + x^{o}$$
$$\Rightarrow \quad x^{o} = 100^{o}$$

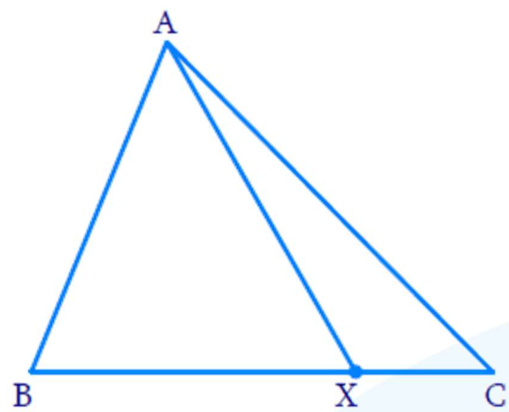
S13. We have:

$$\angle ABD = \frac{1}{2}\times\angle AOD = 80^{o}$$
$$\Rightarrow \quad \angle ABC = 180^{o} - 80^{o} = 100^{o}$$
$$\Rightarrow \quad \angle APC(\text{reflex}) = 2\times\angle ABC$$
$$= 200^{o}$$
$$\Rightarrow \quad \angle APC = 360^{o} - \angle APC(\text{reflex})$$
$$= 160^{o}$$

Miscellaneous

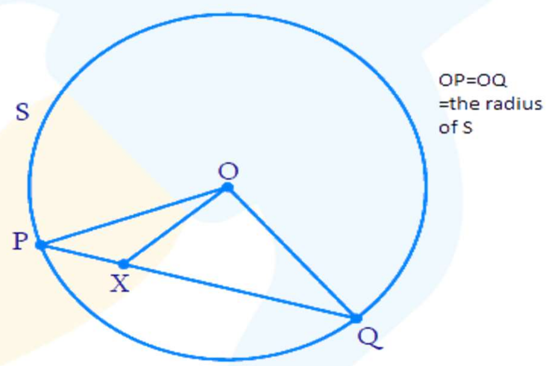
S14. Your first response might be: the truth of this statement is so obvious. That's true, but one aspect of geometry is constructing proofs by logical deduction. Let's see if we can do that in this case.

We will make use of the following result, which we have already proved in our discussion on triangles. Consider a ΔABC, and let X by any point on BC. Join AX:



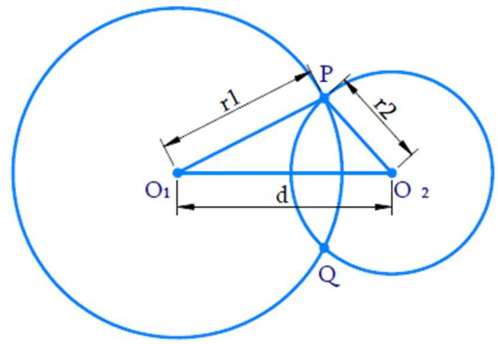
Then, AX is smaller than at least one of AB or AC. If you have forgotten this, you are urged to review the relevant material once again.

Now, coming back to our original problem, PQ is a chord of S. Let X be any point PQ, and let O be the center of S:



Using our earlier result, OX must be less than at one of OP or OQ, In fact, since OP = OQ, OX is less than both. Thus, the distance of X from O is less than the radius of S, which means that X lies inside S.

S15. Consider the following figure:



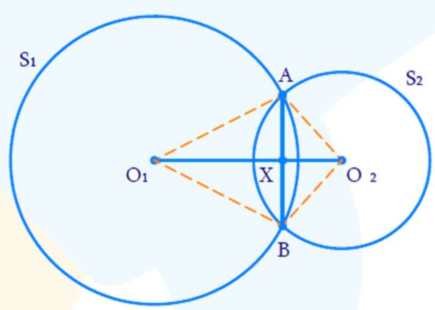
Observe ΔO_1PO_2, the lengths of whose sides are r_1, r_2 and d. According to the triangle inequality:

- the sum of two sides of a triangle is greater than the third, so that $r_1 + r_2 > d$.
- the difference of two sides of a triangle is less than the third, so that $|r_1 - r_2| < d$.

Combining these two results, we have:

$$|r_1 - r_2| < d < r_1 + r_2$$

S16. The following figure shows two circles intersecting at A and B, so that AB is the common chord:



O_1O_2 intersects AB at X we have to prove that X is the mid-point of AB, and that $O_1O_2 \perp AB$.

Compare ΔO_1AO_2 with ΔO_1BO_2:

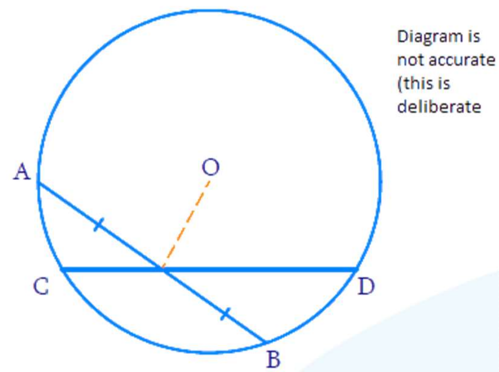
- $O_1A = O_1B$ (radii of S_1)
- $O_2A = O_2B$ (radii of S_2)
- $O_1O_2 = O_1O_2$ (common)

Thus, $\Delta O_1AO_2 \cong \Delta O_1BO_2$ by the SSS criterion. This means that $\angle AO_1X = \angle BO_1X$.

- $O_1A = O_1B$ (radii of S_1)
- $\angle AO_1X = \angle BO_1X$ (proved above)
- $O_1X = O_1X$

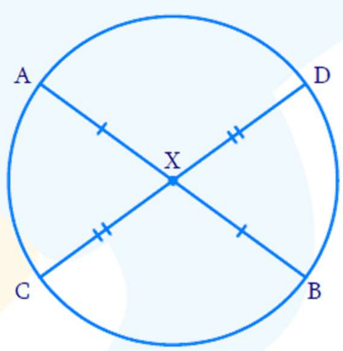
Clearly, $\Delta AO_1X \cong \Delta BO_1X$ by the SAS criterion. This means that AX = BX and $\angle O_1XA = \angle O_1XB = 90^o$. Thus, O_1O_2 bisects AB at right angles.

S17. (a) Suppose that AB and CD are two chords or a circle which bisects each other at X. We also suppose that X is a point different from the center O of the circle:



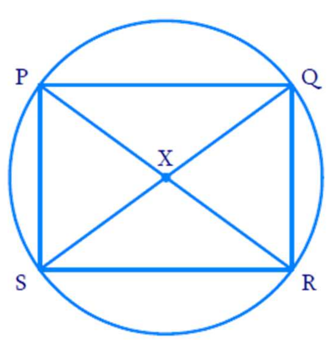
Now, since X is the mid-point of AB, OX must be perpendicular to AB. Similarly, since X is **also** the mid-point of CD, OX must also be perpendicular to CD. Thus, OX is perpendicular to both AB and CD, which means that AB must be parallel to CD.

However, that is clearly not the case, as AB and CD intersect at X. Hence, our original assumption of taking X to be a point different from O must be incorrect. X itself is the point O, that is, the center of the circle:



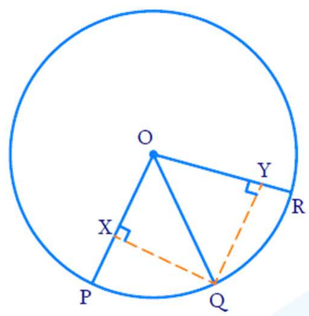
We conclude that two chords of a circle can bisect each other **only if** they are both diameters of the circles.

(b) The following figure shows that the vertices of parallelogram PQRS lie on a circle:



Since PR and QS are diagonals of a parallelogram, they must bisect each other at X. From part-(a), this means that PR and QS are diameters of this circle, and X is its center. Note that PQRS must be a rectangle (can you see how?).

S18. In the following figure, O is the center of the circle, and $\overparen{PQ} = \overparen{QR}$, which means that $\angle POQ = \angle QOR$. Also QX and QY are perpendiculars from Q onto OP and QR respectively:



Compare ΔOQX with ΔOQY :

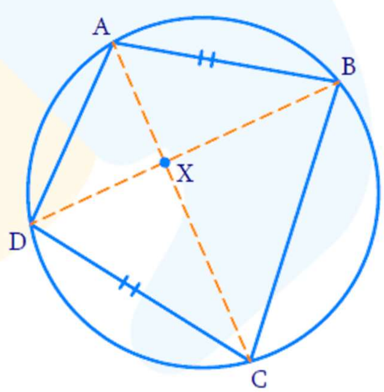
* $OQ = OQ$

* $\angle QOX = \angle QOY$

* $\angle QXO = \angle QYO = 90^o$

Thus, $\Delta OBX \cong \Delta OQY$ by the ASA criterion, so that QX = QY. This means that Q is equidistant from OP and OR.

S19. The fact that ABCD is inscribed in a circle is very significant, since every quadrilateral cannot be inscribed in a circle. Consider the following figure:



Compare ΔABX with ΔDCX :

* AB = DC (given)

* $\angle AXB = \angle DXC$ (vertically opposite angles)

* $\angle ABX = \angle DCX$ (angles subtended by the same chord AD on the circumference)

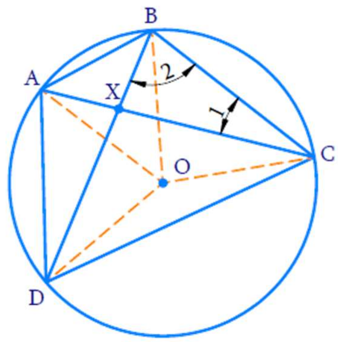
Thus, $\Delta ABX \cong \Delta DCX$, so that:

$\Rightarrow \; AX = DX \text{ and } XB = XC$

$\Rightarrow \; AX + XC = DX + XB$

$\Rightarrow \; AC = BD$

S20. Consider the following figure:



We need to show that

* $\angle AOB = 2\angle 1$
* $\angle COD = 2\angle 2$

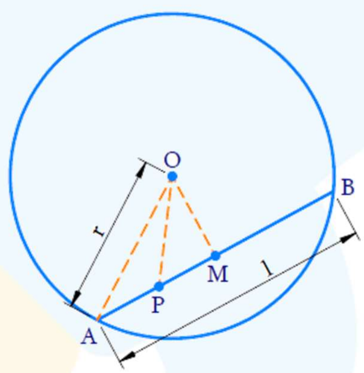
Thus,

$$\angle AOB + \angle COD = 2(\angle 1 + \angle 2)$$

However, if you consider ΔBXC, it is immediately evident that $\angle 1 + \angle 2 = 90^{\circ}$. Thus,

$$\angle AOB + \angle COD = 2 \times 90^{\circ} = 180^{\circ}$$

S21. Consider the following figure. AB slides around the circle but its lengths is always *l*. OA = r is the fixe radius of the circle. M is the mid-point of AB, and PM = d is fixed:

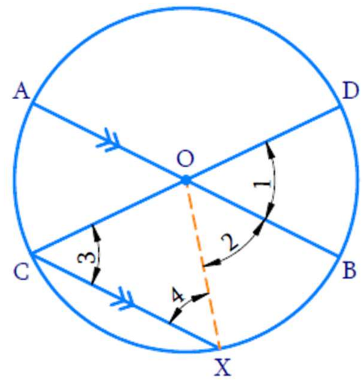


Note using the Pythagoras theorem that:

* $OM = \sqrt{OA^2 - AM^2} = \sqrt{r^2 - \frac{\ell^2}{4}}$ is fixed
* $OP = \sqrt{OM^2 + PM^2} = \sqrt{r^2 - \frac{\ell^2}{4} + d^2}$ is thus fixed.

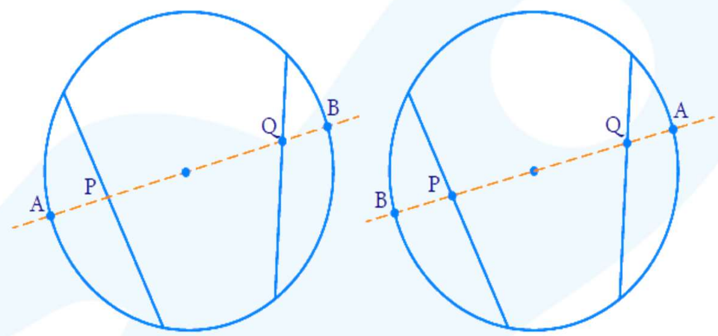
We conclude that as the chord slides around the circle, the distance of P from O remains fixed. The locus of P is a circle concentric with the original circle (that is, the center of this circle is also O).

S22. Consider the following figure:



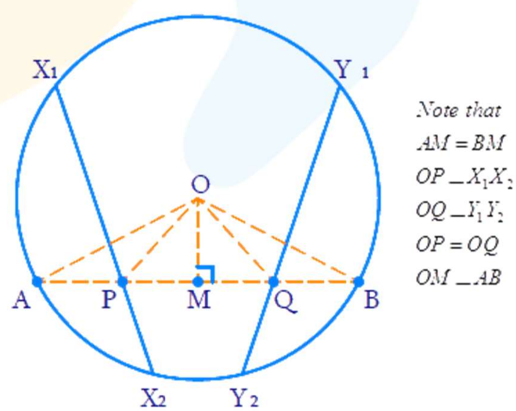
From the observations made in the figure, $\angle 1 = \angle 2$, which means that $\widehat{BB} = \widehat{BX}$. Thus, B is the mid-point of $\widehat{DBX}$.

S23. Two possibilities arise, as shown in the following figure:



The relative positioning of A and B with respect to P and Q is different in these case.

However, in the cases, AP = BQ will hold true. Let us prove this for the first case. Let O be the center of the circle. Let M be the mid-point of AB:



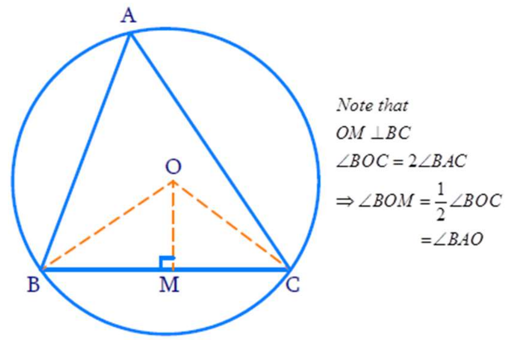
From the observations made in the figure, $\Delta OPM \cong \Delta OQM$, and so, P = QM. Also, since AM = BM, we have:

$$AM - PM = BM - QM$$

$$\Rightarrow \quad AP = BQ$$

The proof for the other case is similar and is left to you as an exercise.

S24. Consider the following figure. The fact that $\angle ABC$ is acute angled means that the circumcircle O will lie inside $\angle ABC$ (why?) . M is the mid-point of BC:

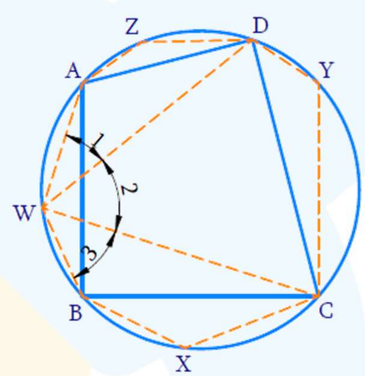


Using the observation made in the figure, we have:

$$\angle OBC = 90^\circ - \angle BOM = 90^\circ - \angle BAC$$
$$\Rightarrow \quad \angle OBC + \angle BAC = 90^\circ$$

Thus, the two angles are complementary.

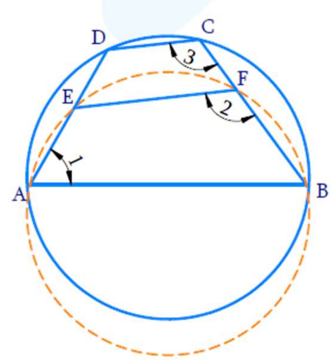
S25. Consider the following figure. We need to show that the sum of ∠W, ∠X, ∠Y, ∠Z is 540º. We have joined W to C and W to D.



We make the following observations:

$\angle 1 = \angle 3$, and

S26. Consider the following figure:

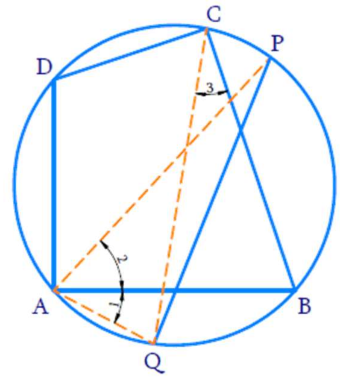


Since ABFE is cyclic,

∠1 + ∠2 = 180º

This means that ∠2 = ∠3. Since these are equal corresponding angles, EF || DC.

S27. Observe the following figure. We show that ∠PAQ = 90º:



We have:

$$\angle 2 + \angle 3 = \frac{1}{2}(\angle DAB + \angle DCB)$$

$$= \frac{1}{2} \times 180^{\circ} = 50^{\circ}$$

Also, ∠1 = ∠3, and so:

∠1 + ∠2 = 90º

Thus, PQ subtends a right angle at the circumference, which means that PQ is a diameter of the circle.

S28. We have:

∠D = 180º – ∠B = 72º

⇒ ∠DAQ = ∠D = 72º

⇒ ∠BAD = 180º = 72º

= 180º – (∠DAQ + ∠QAP)

= 80º

⇒ ∠BCD = 180º – ∠BAD

= 100º